

This article was downloaded by: [Siauliu University Library]

On: 17 February 2013, At: 07:04

Publisher: Taylor & Francis

Informa Ltd Registered in England and Wales Registered Number: 1072954

Registered office: Mortimer House, 37-41 Mortimer Street, London W1T 3JH, UK



## Advanced Composite Materials

Publication details, including instructions for authors and subscription information:

<http://www.tandfonline.com/loi/tacm20>

### Stacking sequence optimizations using modified global response surface in lamination parameters

Akira Todoroki , Kentarou Suenaga & Yoshinobu Shimamura

Version of record first published: 02 Apr 2012.

To cite this article: Akira Todoroki , Kentarou Suenaga & Yoshinobu Shimamura (2003): Stacking sequence optimizations using modified global response surface in lamination parameters , Advanced Composite Materials, 12:1, 35-55

To link to this article: <http://dx.doi.org/10.1163/156855103322320365>

PLEASE SCROLL DOWN FOR ARTICLE

Full terms and conditions of use: <http://www.tandfonline.com/page/terms-and-conditions>

This article may be used for research, teaching, and private study purposes. Any substantial or systematic reproduction, redistribution, reselling, loan, sub-licensing, systematic supply, or distribution in any form to anyone is expressly forbidden.

The publisher does not give any warranty express or implied or make any representation that the contents will be complete or accurate or up to date. The accuracy of any instructions, formulae, and drug doses should be independently verified with primary sources. The publisher shall not be liable for any loss, actions, claims, proceedings, demand, or costs or

damages whatsoever or howsoever caused arising directly or indirectly in connection with or arising out of the use of this material.

## Stacking sequence optimizations using modified global response surface in lamination parameters

AKIRA TODOROKI\*, KENTAROU SUENAGA  
and YOSHINOBU SHIMAMURA

*Department of Mechanical Sciences and Engineering, Tokyo Institute of Technology,  
2-12-1 Ohokayama, Meguro, Tokyo 152-8552, Japan*

Received 19 August 2002; accepted 21 January 2003

**Abstract**—The present paper proposes a new modified global response surface method using lamination parameters to overcome the problem of how to create zoomed response surfaces. The present study also proposes a new low-cost rule for optimality. The new optimality rule is better for the search-all-branches method for relatively thin laminated composite structures. The new method is applied to a stacking sequence optimization problem of a composite plate with hat-type stiffeners for maximizations of buckling load. FEM analyses are conducted to create a global response surface and the optimal stacking sequence is obtained by using the proposed method. As a result, the new method successfully obtained the true optimal stacking sequence and reduced the computational cost.

**Keywords:** Composites; stacking sequence; response surface; design of experiment; optimization.

### 1. INTRODUCTION

Stacking sequence optimizations for laminated composites are indispensable for designs of composite structures. One stacking sequence optimization method is a graphical design method with lamination parameters proposed by Miki [1] and Fukunaga and Chou [2], which employs lamination parameters as continuous design variables and obtains an optimal stacking sequence of an angle ply laminate, such as  $\pm\theta$ . For practical composite structures, however, available fiber angles are limited to a small set of  $0^\circ$ ,  $\pm 45^\circ$ , and  $90^\circ$  plies by lack of experimental data and the complicated fabrication process. These limitations give rise to stacking sequence optimizations that are combinatorial optimization problems with some constraints. Susuki [3] proposed a ranking method for strength optimizations of laminated composites for in-plane loadings. For general stacking sequence opti-

---

\*To whom correspondence should be addressed. E-mail: atodorok@ginza.mes.titech.ac.jp

mizations, genetic algorithms (GAs) are becoming popular [4–13], because GAs are generally admitted to be effective for combinatorial optimizations. For stacking sequence optimizations, however, there are several combinatorial constraints that are normally difficult to implement in GAs. For example, to prevent a tension-shear coupling effect, the number of angle plies must be balanced. To prevent a large matrix cracking effect, stacking of more than four plies of identical fiber angles must be avoided. Penalty methods to implement these combinatorial constraints reduce GA performance; for this, a recessive-gene-like repair strategy has been proposed by some authors [14, 15].

The GAs require high computational cost because algorithms repeat evaluations for numerous genes. Yamazaki has proposed a two-stage method to reduce computational cost [7]: in the first stage, optimal lamination parameters are obtained with a mathematical programming method assuming lamination parameters are continuous independent variables; then, the most contiguous practical laminate is selected with a GA in the second stage. This method provides improved optimal results, but it does not always give a truly optimal result because in-plane lamination parameters and out-of-plane lamination parameters are not independent of each other for practical laminates. The authors proposed a new GA for stacking sequence optimizations using response surface approximations in lamination parameters [13]; that method excellently reduces computational cost. In that method, GA is employed to search an optimal stacking sequence, because this is a quite difficult problem with many local optimal points. Of course, in the case of thin laminates, it is easy to search all feasible laminates using response surface approximation of the objective function.

For the stacking sequence optimizations using the GA with the response surface approximations, one of the authors has proposed an optimality check method [16] after obtaining an optimal laminate from a global response surface. The optimality check method is a proposal of the method to obtain adjacent laminates in lamination parameter spaces, and the method proposed a new optimality check process to know the necessity of making a zoomed response surface near the optimal point obtained from the global response surface.

The previous paper [16] reports that there are 62 adjacent laminates at the maximum near the optimal laminate. Since it requires high cost to evaluate all of the adjacent laminates, it proposed a reduced computational-cost method comprising 38 evaluations. For complicated structures, such as stiffened panels, it requires more computational cost because we have to optimize multiple laminates at the same time, such as a panel and a stiffener. Moreover, it requires solving a difficult problem to find the appropriate zooming region and candidate laminates for a creation of the zoomed response surface.

Although GAs are the appropriate optimization method for complicated problems, it is easy to evaluate all feasible laminates when the laminate is thin and its evaluation cost is negligible: for example, a response surface may reduce the cost. Most aerospace structures have thin skin structures comprising eight or ten plies.

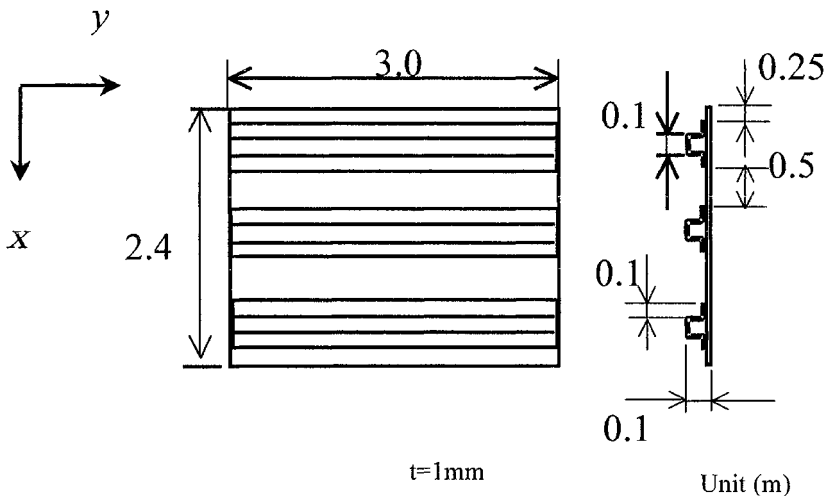
Since the response surface usually comprises simple quadratic polynomials, it is very easy to evaluate all feasible laminates after obtaining the response surface for these thin structures. For these thin structures, the most important thing is to obtain a more precisely fitted response surface.

In the present study, therefore, we focus on the creation of a precise global response surface for optimizations of stacking sequences of thin laminates of complicated structures by searching all feasible laminates. The present paper proposes a more precise global response surface near the optimal laminate, which is obtained after reducing a bias near the optimal laminate. The method is applied to a stacking sequence optimization of a panel with hat-type stiffeners to maximize a buckling load. For this problem, FEM analyses are performed to obtain buckling loads of various types of stacking sequences. After we obtain the modified precise global response surface, all feasible laminates are evaluated to find the real optimal the stacking sequence.

## 2. OPTIMIZATION PROBLEM

### 2.1. Buckling load analysis of composite panel with hat-type stiffeners

The configuration of the simply supported panel with the hat-type panel stiffeners is shown in Fig. 1. The longitudinal length  $y$  is 3 m, and the width  $x$  is 2.4 m. The thickness  $h$  is 1 mm. The height of the hat-type stiffener, the width and the length of the flange of the stiffener are all fixed at 0.1 m. The stiffener is attached on the panel at the points with spacing of 0.5 m, and the endmost stiffener is attached to the point at the distance of 0.25 m from the edge. The material properties used here are taken from T300/5208 ( $E_L = 133$  GPa,  $E_T = 8.7$  GPa,



**Figure 1.** Configuration of hat stiffened laminated structure.

$G_{LT} = 3.2$  GPa,  $\nu_{LT} = 0.26$ ). The stiffened panel is compressed uniformly in the longitudinal direction ( $y$ -direction). The panel and the stiffener are both symmetric laminates of eight plies (the thickness of a ply is 0.125 mm). The available fiber angles for the laminates are limited to  $0^\circ$ ,  $\pm 45^\circ$ , and  $90^\circ$  plies by lack of experimental data and the complicated fabrication process. To obtain stacking sequences of the panel and the stiffener from the four available fiber angles to maximize a buckling load of the stiffened panel is an optimization problem here. For the buckling analyses, a commercially available FEM code ANSYS is employed here. For the FEM analyses, shell type elements are adopted, and the structure is divided into 1560 elements and 5000 grids using an auto-mesh division method of ANSYS.

As previously mentioned, the available fiber angles are limited to four kinds, and more constraints are imposed as follows:

- (1) The number of angle plies ( $\pm 45^\circ$ ) has to be balanced.
- (2) More than four contiguous plies of the same fiber angle must be prevented.
- (3) Anisotropic parameters of  $\gamma$  and  $\delta$  for bending-twisting coupling must be smaller than 0.2 [17].

The constraint number (1) is adopted to eliminate the tension-shear coupling. The constraint number (2) is adopted to prevent large matrix cracking. The constraint number (3) is adopted to reduce the bending-twisting coupling. The  $\delta$  and  $\gamma$  are given as follows.

$$\gamma = \frac{D_{16}}{(D_{11}^3 D_{22})^{1/4}}, \quad \delta = \frac{D_{26}}{(D_{22}^3 D_{11})^{1/4}}. \quad (1)$$

## 2.2. Lamination parameters

Lamination parameters are calculated from the stacking sequence. The in-plane and out-of-plane lamination parameters of a symmetric laminate are given as follows.

$$\left. \begin{aligned} V_1^* &= \frac{1}{h} \int_{-h/2}^{h/2} \cos 2\theta \, dz \\ V_2^* &= \frac{1}{h} \int_{-h/2}^{h/2} \cos 4\theta \, dz \\ V_3^* &= \frac{1}{h} \int_{-h/2}^{h/2} \sin 2\theta \, dz \\ V_4^* &= \frac{1}{h} \int_{-h/2}^{h/2} \sin 4\theta \, dz \end{aligned} \right\}, \quad (2)$$

$$\left. \begin{aligned} W_1^* &= \frac{12}{h^3} \int_{-h/2}^{h/2} \cos 2\theta z^2 dz \\ W_2^* &= \frac{12}{h^3} \int_{-h/2}^{h/2} \cos 4\theta z^2 dz \\ W_3^* &= \frac{12}{h^3} \int_{-h/2}^{h/2} \sin 2\theta z^2 dz \\ W_4^* &= \frac{12}{h^3} \int_{-h/2}^{h/2} \sin 4\theta z^2 dz \end{aligned} \right\}, \quad (3)$$

where  $z$  is the axis in the thickness direction from the middle of the laminate,  $\theta$  is the fiber angle and  $h$  is the thickness of the laminate.

All elements of the stiffness matrix of the laminate can be obtained from these lamination parameters and material properties [18, 19]. From equations (2) and (3), the total number of lamination parameters is fixed to eight for symmetric laminates. For the balanced symmetric laminates comprising four fiber angles, such as  $0^\circ$ ,  $\pm 45^\circ$ , and  $90^\circ$  plies, however,  $V_3^*$ ,  $V_4^*$  and  $W_4^*$  vanish. Since the constraint of the maximum value of the anisotropic parameters  $\delta$  and  $\gamma$  is imposed here,  $W_3^*$  is kept at a small value. This value, therefore, is neglected here. Thus, we have only four lamination parameters as design variables:  $V_1^*$ ,  $V_2^*$ ,  $W_1^*$  and  $W_2^*$ . These four lamination parameters are employed as variables of a response surface approximation for an objective function instead of using fiber angles. This employment reduces the number of variables, and the employment also reduces non-linearity of the objective function, as discussed by Fukunaga [20].

For practical stiffened panels, designers usually have to consider matching of coefficients of linear thermal expansion between the panel and the stiffener to prevent fracture owing to the thermal stresses during curing. In the present study, however, we consider the stacking sequences of the panel and of the stiffeners are independent variables for the simplicity of the problem. In this case, the number of variables is 16 ( $= 8 \text{ plies} \times 2 \text{ laminates}$ ).

### 3. RESPONSE SURFACE METHOD

Response surface methodology is applied to obtain an approximation to a response function in terms of predictor variables [21]. The response surface methodology comprises experimental design, the least-square-error method, and optimizations.

In the present study, response  $y$  is buckling load and variables are four lamination parameters:  $V_1^*$ ,  $V_2^*$ ,  $W_1^*$ , and  $W_2^*$  of each laminate: the panel and the stiffeners. The lamination parameters of the panel are expressed as  $V_{1P}^*$ ,  $V_{2P}^*$ ,  $W_{1P}^*$ , and  $W_{2P}^*$ . The lamination parameters of the stiffeners are expressed as  $V_{1S}^*$ ,  $V_{2S}^*$ ,  $W_{1S}^*$ , and  $W_{2S}^*$ . These eight variables are replaced as follows:  $x_1 = V_{1P}^*$ ,  $x_2 = V_{2P}^*$ ,  $x_3 = W_{1P}^*$ ,  $x_4 = W_{2P}^*$ ,  $x_5 = V_{1S}^*$ ,  $x_6 = V_{2S}^*$ ,  $x_7 = W_{1S}^*$ ,  $x_8 = W_{2S}^*$ . The response surface of quadratic polynomials is as follows:

$$y = \beta_0 + \sum_{i=1}^8 \beta_i x_i + \sum_{i=1, j=1, j \geq i}^8 \beta_{ij} x_i x_j, \quad (4)$$

where replacements of  $x_9 = x_1^2$ ,  $\beta_9 = \beta_{11}$ ,  $x_{10} = x_1 x_2$ ,  $\beta_{10} = \beta_{12}$ ,  $\dots$ ,  $x_{44} = x_8^2$  and  $\beta_{44} = \beta_{88}$ , Equation (4) becomes a simple linear multiple regression of 45 unknown coefficients as follows.

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_5 + \beta_6 x_6 + \beta_7 x_7 \\ + \beta_8 x_8 + \dots + \beta_{43} x_{43} + \beta_{44} x_{44}. \quad (5)$$

All coefficients can be obtained from the simple well-known least-square-error method.

Let us consider the case where we perform  $n$  calculations of responses ( $n > k$  or  $n = k$ , where  $k$  is the number of variables in equation (5)). The number of unknown coefficients is  $p = k + 1$ . The matrix form of the linear multiple regression model is rewritten as

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}, \quad (6)$$

$$\mathbf{y} = \begin{Bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{Bmatrix}, \quad \mathbf{X} = \begin{bmatrix} 1 & x_{11} & x_{12} & \cdots & x_{1k} \\ 1 & x_{21} & x_{22} & \cdots & x_{2k} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_{n1} & x_{n2} & \cdots & x_{nk} \end{bmatrix},$$

$$\boldsymbol{\beta} = \begin{Bmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_k \end{Bmatrix}, \quad \boldsymbol{\varepsilon} = \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{Bmatrix},$$

where  $\mathbf{y}$  is a response vector,  $\mathbf{X}$  is a variable matrix,  $\boldsymbol{\beta}$  is a coefficient vector, and  $\boldsymbol{\varepsilon}$  is an error vector. The unbiased estimator  $\mathbf{b}$  of coefficient  $\boldsymbol{\beta}$  is obtained using the least-square-error method.

$$\mathbf{b} = (\mathbf{X}^T \mathbf{X})^{-1} \mathbf{X}^T \mathbf{y}. \quad (7)$$

Multiple sets of responses and variables of different points are required to obtain coefficients using the least-square-error-method. To reduce variance of coefficients on the limited number of calculations or experiments, we must carefully select experimental or calculation points. This process is called design of experiments. In the present study, a previously proposed D-optimal laminate is employed [22]: a set of 90 feasible laminates selected from all feasible laminates of  $8 = 2$  (panel and stiffener)  $\times 4$  (half of 8 plies) plies of balanced symmetric laminates with the D-optimality.

The D-optimal laminates are selected from all possible laminate combinations of 8 plies: 1681 laminates. In the present study, trinary numbers are employed as codes of ply angles as in Harrison *et al.* [8]; the number of entire laminates



is, therefore,  $3^8 = 6561$ . From the set of 6561, the feasible set of laminates that satisfy the constraints mentioned previously comprises laminates of 1681. The set of feasible laminates of 1681 is a set of candidate laminates. From the set of candidate laminates, the best set of laminates of 90 is selected with D-optimality. The D-optimality is a computer-aided design of experiments to reduce variants and co-variants maximizing a determinant of a moment matrix defined as follows [21].

$$M = X^T X / n. \quad (8)$$

Since the number of unknown coefficients is 45 in the present study, and more than twice the number of unknown coefficients is required empirically, 90 laminates were selected here from feasible laminates with D-optimality. For the objective function of D-optimality, D-efficiency  $D_{eff}$  is used; the definition of  $D_{eff}$  is shown as follows.

$$D_{eff} = \frac{(\det[\mathbf{X}^T \mathbf{X}])^{1/p}}{k}. \quad (9)$$

Normalizing all variables  $x$  from  $-1$  to  $1$ , the value of  $D_{eff}$  must be from  $0$  to  $1$ . The highest value of  $D_{eff}$  gives the best experiment design. The obtained set of laminates with the D-optimality is shown in Table 1. All of the selected laminates are plotted in the lamination parameters as shown in Fig. 2. Figures 2a and 2b show the selected laminates of the panel, and Figs 2c and 2d show the selected laminates of the stiffeners.  $D_{eff}$  of the set of selected laminated is 4.8%. As shown in Fig. 2, the values of the in-plane lamination parameters of the selected laminates of the panel and stiffeners are almost equally spacing, and the values of the out-of-plane laminates of these selected laminates are distributed on the boundary. As mentioned in Ref. [16], the selected set of laminates can be applied to other laminates of other thickness that are thicker than 8 plies (see Ref. [16]). In Table 1, the computed buckling load results (N) of FEM analyses are also listed.

When we make a response surface, higher order terms that worsen the adjusted coefficient of multiple determination  $R_{ad}^2$  are deleted from the response surface using F-statistics. The reason why we adopted the quadratic polynomials is that the buckling load response is not so complicated in the lamination parameters as shown in references [13, 16].

## 4. APPLICATION OF MODIFIED RESPONSE SURFACE

### 4.1. Zoomed response surface

For a single panel composite structure, it is not difficult to obtain a more precise response surface in the lamination parameters as mentioned in our previous paper [16]. In the previous paper, a provisional optimal laminate is selected using the GA with a global response surface, and adjacent laminates around the provisional optimal laminate are prepared for estimations of optimality. After the optimality check, a zoomed response surface is made when the optimality of the provisional

**Table 1.**  
Stacking sequences obtained by D-optimal design of experiments

Plate	Stiffener	FEM	V <sub>1P</sub>	V <sub>2P</sub>	W <sub>1P</sub>	W <sub>2P</sub>	V <sub>1S</sub>	V <sub>2S</sub>	W <sub>1S</sub>	W <sub>2S</sub>
[0/0/0/0]s	[0/0/0/90]s	853.0489	1.0000	1.0000	1.0000	1.0000	0.5000	1.0000	0.9688	1.0000
[0/0/0/0]s	[0/0/45/-45]s	824.6305	1.0000	1.0000	1.0000	1.0000	0.5000	0.0000	0.8750	0.7500
[0/0/0/0]s	[0/90/90/90]s	900.6832	1.0000	1.0000	1.0000	1.0000	-0.5000	1.0000	0.1563	1.0000
[0/0/0/0]s	[45/-45/0/0]s	825.7881	1.0000	1.0000	1.0000	1.0000	0.5000	0.0000	0.1250	-0.7500
[0/0/0/0]s	[45/-45/45/-45]s	802.7007	1.0000	1.0000	1.0000	1.0000	0.0000	-1.0000	0.0000	-1.0000
[0/0/0/0]s	[90/90/0/0]s	877.3571	1.0000	1.0000	1.0000	1.0000	0.0000	1.0000	-0.7500	1.0000
[0/0/0/0]s	[90/90/45/-45]s	902.0763	1.0000	1.0000	1.0000	1.0000	-0.5000	0.0000	-0.8750	0.7500
[0/0/0/0]s	[90/90/90/90]s	838.7344	1.0000	1.0000	1.0000	1.0000	-1.0000	1.0000	-1.000	1.0000
[0/0/0/90]s	[0/45/-45/90]s	778.804	0.5000	1.0000	0.9688	1.0000	0.0000	0.0000	0.5625	0.1875
[0/0/45/-45]s	[0/0/0/0]s	717.9394	0.5000	0.0000	0.8750	0.7500	1.0000	1.0000	1.0000	1.0000
[0/0/45/-45]s	[0/90/45/-45]s	938.4064	0.5000	0.0000	0.8750	0.7500	0.0000	0.0000	0.2813	0.7500
[0/0/45/-45]s	[45/-45/45/-45]s	831.1063	0.5000	0.0000	0.8750	0.7500	0.0000	-1.0000	0.0000	-1.0000
[0/0/45/-45]s	[45/-45/90/90]s	964.9412	0.5000	0.0000	0.8750	0.7500	-0.5000	0.0000	-0.1250	-0.7500
[0/0/45/-45]s	[90/90/0/0]s	905.2157	0.5000	0.0000	0.8750	0.7500	0.0000	1.0000	-0.7500	1.0000
[0/0/45/-45]s	[90/90/90/90]s	917.4363	0.5000	0.0000	0.8750	0.7500	-1.0000	1.0000	-1.0000	1.0000
[0/0/90/90]s	[0/0/0/0]s	859.9	0.0000	1.0000	0.7500	1.0000	1.0000	1.0000	1.0000	1.0000
[0/0/90/90]s	[0/0/45/-45]s	937.2124	0.0000	1.0000	0.7500	1.0000	0.5000	0.0000	0.8750	0.7500
[0/0/90/90]s	[45/-45/90/90]s	965.9493	0.0000	1.0000	0.7500	1.0000	-0.5000	0.0000	-0.1250	-0.7500
[0/0/90/90]s	[90/0/0/90]s	940.2937	0.0000	1.0000	0.7500	1.0000	0.0000	1.0000	-0.1875	1.0000
[0/0/90/90]s	[90/45/-45/0]s	952.7306	0.0000	1.0000	0.7500	1.0000	0.0000	0.0000	-0.5625	0.1875
[0/45/-45/0]s	[0/0/0/0]s	927.7946	0.5000	0.0000	0.5938	0.1875	1.0000	1.0000	1.0000	1.0000
[0/45/-45/90]s	[0/0/90/90]s	1124.183	0.0000	0.0000	0.5625	0.1875	0.0000	1.0000	0.7500	1.0000
[0/45/-45/90]s	[45/-45/0/0]s	1066.629	0.0000	0.0000	0.5625	0.1875	0.5000	0.0000	0.1250	-0.7500
[0/4B/-45/90]s	[90/90/45/-45]s	1142.083	0.0000	0.0000	0.5625	0.1875	-0.5000	0.0000	-0.8750	0.7500
[0/90/45/-45]s	[0/45/-45/0]s	1142.933	0.0000	0.0000	0.2813	0.7500	0.5000	0.0000	0.5938	0.1875
[0/90/90/0]s	[45/90/90/-45]s	1205.694	0.0000	1.0000	0.1875	1.0000	-0.5000	0.0000	-0.4063	-0.1875
[0/90/90/0]s	[90/0/45/-45]s	1216.569	0.0000	1.0000	0.1875	1.0000	0.0000	0.0000	-0.2813	0.7500
[0/90/90/0]s	[90/90/0/0]s	1156.588	0.0000	1.0000	0.1875	1.0000	0.0000	1.0000	-0.7500	1.0000

**Table 1.**  
(Continued)

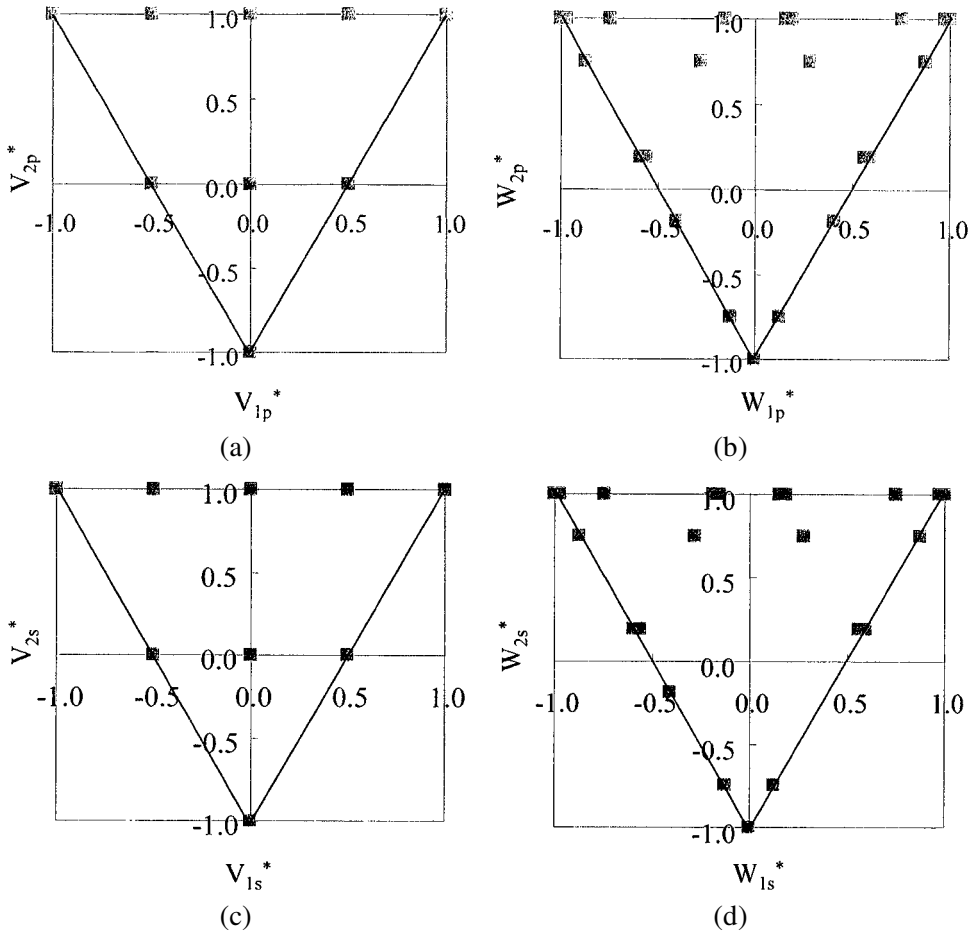
Plate	Stiffener	FEM	V <sub>1P</sub>	V <sub>2P</sub>	W <sub>1P</sub>	W <sub>2P</sub>	V <sub>1S</sub>	V <sub>2S</sub>	W <sub>1S</sub>	W <sub>2S</sub>
[0/90/90/90]s	[0/0/90/90]s	1204.812	-0.5000	1.0000	0.1563	1.0000	0.0000	1.0000	0.7500	1.0000
[0/90/90/90]s	[45/-45/0/0]s	1147.369	-0.5000	1.0000	0.1563	1.0000	0.5000	0.0000	0.1250	-0.7500
[0/98/90/90]s	[45/-45/45/-45]s	1117.738	-0.5000	1.0000	0.1563	1.0000	0.0000	-1.0000	0.0000	-1.0000
[0/90/90/90]s	[90/90/90/90]s	1060.344	-0.5000	1.0000	0.1563	1.0000	-1.0000	1.0000	-1.0000	1.0000
[45/0/0/-45]s	[0/45/-45/90]s	1182.365	0.5000	0.0000	0.4063	-0.1875	0.0000	0.0000	0.5625	0.1875
[45/0/0/-45]s	[90/0/0/90]s	1172.082	0.5000	0.0000	0.4063	-0.1875	0.0000	1.0000	-0.1875	1.0000
[45/-45/0/0]s	[0/0/0/0]s	1093.124	0.5000	0.0000	0.1250	-0.7500	1.0000	1.0000	1.0000	1.0000
[45/-45/0/0]s	[0/90/45/-45]s	1448.012	0.5000	0.0000	0.1250	-0.7500	0.0000	0.0000	0.2813	0.7500
[45/-45/0/0]s	[0/90/90/90]s	1547.275	0.5000	0.0000	0.1250	-0.7500	-0.5000	1.0000	0.1563	1.0000
[45/-45/0/0]s	[45/-45/0/0]s	1232.337	0.5000	0.0000	0.1250	-0.7500	0.5000	0.0000	0.1250	-0.7500
[45/-45/0/0]s	[45/-45/45/-45]s	1183.98	0.5000	0.0000	0.1250	-0.7500	0.0000	-1.0000	0.0000	-1.0000
[45/-45/0/0]s	[45/-45/90/90]s	1495.314	0.5000	0.0000	0.1250	-0.7500	-0.5000	0.0000	-0.1250	-0.7500
[45/-45/0/0]s	[90/90/0/0]s	1376.737	0.5000	0.0000	0.1250	-0.7500	0.0000	1.0000	-0.7500	1.0000
[45/-45/0/0]s	[90/90/45/-45]s	1439.049	0.5000	0.0000	0.1250	-0.7500	-0.5000	0.0000	-0.8750	0.7500
[45/-45/45/-45]s	[0/0/45/-45]s	972.502	0.0000	-1.0000	0.0000	-1.0000	0.5000	0.0000	0.8750	0.7500
[45/-45/45/-45]s	[0/0/90/90]s	1184.022	0.0000	-1.0000	0.0000	-1.0000	0.0000	1.0000	0.7500	1.0000
[45/-45/45/-45]s	[45/-45/0/0]s	906.5125	0.0000	-1.0000	0.0000	-1.0000	0.5000	0.0000	0.1250	-0.7500
[45/-45/45/-45]s	[45/-45/45/-45]s	838.9806	0.0000	-1.0000	0.0000	-1.0000	0.0000	-1.0000	0.0000	-1.0000
[45/-45/45/-45]s	[90/0/0/0]s	1003.667	0.0000	-1.0000	0.0000	-1.0000	0.5000	1.0000	-0.1563	1.0000
[45/-45/45/-45]s	[90/90/45/-45]s	1074.749	0.0000	-1.0000	0.0000	-1.0000	-0.5000	0.0000	-0.8750	0.7500
[45/-45/45/-45]s	[90/90/90/90]s	1127.539	0.0000	-1.0000	0.0000	-1.0000	-1.0000	1.0000	-1.0000	1.0000
[45/-45/90/90]s	[0/0/0/90]s	1284.801	-0.5000	0.0000	-0.1250	-0.7500	0.5000	1.0000	0.9688	1.0000
[45/-45/90/90]s	[0/45/-45/0]s	1278.33	-0.5000	0.0000	-0.1250	-0.7500	0.5000	0.0000	0.5938	0.1875
[45/-45/90/90]s	[0/90/90/90]s	1352.64	-0.5000	0.0000	-0.1250	-0.7500	-0.5000	1.0000	0.1563	1.0000
[45/-45/90/90]s	[45/-45/45/-45]s	1232.258	-0.5000	0.0000	-0.1250	-0.7500	0.0000	-1.0000	0.0000	-1.0000
[45/-45/90/90]s	[45/-45/90/90]s	1341.383	-0.5000	0.0000	-0.1250	-0.7500	-0.5000	0.0000	-0.1250	-0.7500
[45/-45/90/90]s	[90/0/0/0]s	1218.029	-0.5000	0.0000	-0.1250	-0.7500	0.5000	1.0000	-0.1563	1.0000
[45/-45/90/90]s	[90/90/45/-45]s	1289.187	-0.5000	0.0000	-0.1250	-0.7500	-0.5000	0.0000	-0.8750	0.7500
[45/-45/90/90]s	[90/90/90/0]s	1228.906	-0.5000	0.0000	-0.1250	-0.7500	-0.5000	1.0000	-0.9688	1.0000

**Table 1.**  
(Continued)

Plate	Stiffener	FEM	$V_{1P}$	$V_{2P}$	$W_{1P}$	$W_{2P}$	$V_{1S}$	$V_{2S}$	$W_{1S}$	$W_{2S}$
[45/90/90/-45]s	[0/0/0/0]s	862.0976	-0.5000	0.0000	-0.4063	-0.1875	1.0000	1.0000	1.0000	1.0000
[45/90/90/-45]s	[90/45/-45/0]s	929.8446	-0.5000	0.0000	-0.4063	-0.1875	0.0000	0.0000	-0.5625	0.1875
[90/0/0/0]s	[0/0/45/-45]s	1292.393	0.5000	1.0000	-0.1563	1.0000	0.5000	0.0000	0.8750	0.7500
[90/0/0/0]s	[45/-45/90/90]s	1375.369	0.5000	1.0000	-0.1563	1.0000	-0.5000	0.0000	-0.1250	-0.7500
[90/0/0/0]s	[90/0/0/0]s	1283.064	0.5000	1.0000	-0.1563	1.0000	0.5000	1.0000	-0.1563	1.0000
[90/0/0/0]s	[90/45/-45/90]s	1335.866	0.5000	1.0000	-0.1563	1.0000	-0.5000	0.0000	-0.5938	0.1875
[90/0/45/-45]s	[0/90/90/90]s	1318.185	0.0000	0.0000	-0.2813	0.7500	-0.5000	1.0000	0.1563	1.0000
[90/0/45/-45]s	[45/-45/0/0]s	1107.195	0.0000	0.0000	-0.2813	0.7500	0.5000	0.0000	0.1250	-0.7500
[90/0/45/-45]s	[90/45/-45/0]s	1180.407	0.0000	0.0000	-0.2813	0.7500	0.0000	0.0000	-0.5625	0.1875
[90/0/45/-45]s	[90/90/45/-45]s	1200.839	0.0000	0.0000	-0.2813	0.7500	-0.5000	0.0000	-0.8750	0.7500
[90/45/-45/0]s	[0/45/-45/0]s	997.1202	0.0000	0.0000	-0.5625	0.1875	0.5000	0.0000	0.5938	0.1875
[90/45/-45/0]s	[0/90/90/0]s	1169.95	0.0000	0.0000	-0.5625	0.1875	0.0000	1.0000	0.1875	1.0000
[90/45/-45/0]s	[45/90/90/-45]s	1146.155	0.0000	0.0000	-0.5625	0.1875	-0.5000	0.0000	-0.4063	-0.1875
[90/45/-45/90]s	[0/90/45/-45]s	1080.307	-0.5000	0.0000	-0.5938	0.1875	0.0000	0.0000	0.2813	0.7500
[90/90/0/0]s	[0/0/90/90]s	1170.501	0.0000	1.0000	-0.7501	1.0000	0.0000	1.0000	0.7500	1.0000
[90/90/0/0]s	[45/-45/0/0]s	1026.458	0.0000	1.0000	-0.7500	1.0000	0.5000	0.0000	0.1250	-0.7500
[90/90/0/0]s	[90/90/45/-45]s	1063.75	0	1	-0.75	1	-0.5	0	-0.875	0.75
[90/90/0/0]s	[90/90/90/90]s	1006.615	0	1	-0.75	1	-1	1	-1	1

**Table 1.**  
(Continued)

Plate	Stiffener	FEM	$V_{1P}$	$V_{2P}$	$W_{1P}$	$W_{2P}$	$V_{1S}$	$V_{2S}$	$W_{1S}$	$W_{2S}$
[90/90/45/-45]s	[0/0/0/0]s	699.3112	-0.5	0	-0.875	0.75	1	1	1	1
[90/90/45/-45]s	[0/0/45/45]s	786.3701	-0.5	0	-0.875	0.75	0.5	0	0.875	0.75
[90/90/45/-45]s	[0/45/-45/90]s	830.58	-0.5	0	-0.875	0.75	0	0	0.5625	0.1875
[90/90/45/-45]s	[45/-45/45/-45]s	683.2256	-0.5	0	-0.875	0.75	0	-1	0	1
[90/90/45/-45]s	[45/-45/90/90]s	845.294	-0.5	0	-0.875	0.75	-0.5	0	-0.125	-0.75
[90/90/45/-45]s	[90/90/0/0]s	773.7312	-0.5	0	-0.875	0.75	0	-1	-0.75	1
[90/90/45/-45]s	[90/90/90/90]s	760.358	-0.5	0	-0.875	0.75	-1	1	-1	1
[90/90/90/0]s	[45/-45/45/-45]s	720.158	-0.5	1	-0.96875	1	0	-1	0	-1
[90/90/90/90]s	[0/0/0/0]s	661	-1	1	-1	1	1	1	1	1
[90/90/90/90]s	[0/0/45/-45]s	751.471	-1	1	-1	1	0.5	0	0.875	0.75
[90/90/90/90]s	[0/90/90/90]s	747.67	-1	1	-1	1	-0.5	1	0.15625	1
[90/90/90/90]s	[45/-45/0/0]s	704.847	-1	1	-1	1	0.5	0	0.125	-0.75
[90/90/90/90]s	[45/-45/90/90]s	724.1198	-1	1	-1	1	-0.5	0	-0.125	-0.75
[90/90/90/90]s	[90/90/0/0]s	649.47	-1	1	-1	1	0	1	-0.75	1
[90/90/90/90]s	[90/90/45/-45]s	673.27	-1	1	-1	1	-0.5	0	-0.875	0.75



**Figure 2.** Distribution of D-optimal laminates (a) In-plane lamination parameters of the plate. (b) Out-of-plane lamination parameters of the plate. (c) In-plane lamination parameters of the stiffener. (d) Out-of-plane lamination parameters of the stiffener.

optimal laminate is rejected. Detail of the optimality check method is mentioned in our previous paper [16]. The provisional optimal laminate has eight adjacent laminates in the in-plane lamination parameters at the maximum, and it has six adjacent laminates in the out-of-plane lamination parameters at the maximum. When we evaluate all of these adjacent laminates, the maximum number of the adjacent laminates is 62 ( $= 6 + 8 + 8 \times 6$ ). Since it requires computationally high cost to evaluate all of these laminates, we reduced the number of adjacent laminates to evaluate using the global response surface. Obviously inferior laminates are selected using the global response surface, and the inferior laminates are deleted from the list of adjacent laminates. The process in the previous paper is summarized as follows.

- (1) A global response surface is created from responses of laminates selected with the D-optimal laminates; the provisional optimal laminate is selected with GA,

and a true response of the provisional optimal laminate is obtained with FEM analysis ( $F_{RS}$ ).

- (2) FEM analyses are performed for all six adjacent laminates to the provisional optimal laminate obtained by fixing in-plane lamination parameters to those of the provisional optimal laminate ( $F_{n0j}$ ).
- (3) If  $\xi F_{RS} < F_{n0j}$ , a zoomed response surface is created, and other adjacent laminates in in-plane lamination parameters are compared. All adjacent laminates in in-plane lamination parameter coordinates are collected (note that only the number of plies of each fiber angle can be decided here and the stacking sequence is unknown) and the optimal stacking sequence for each fixed in-plane lamination parameters is selected with GA. From these obtained optimal laminates for each fixed in-plane lamination parameters, FEM analyses are performed for four laminates from the top value of the objective function of the response surface ( $F_{ni}$ ).
- (4) If  $\xi F_{RS} > F_{ni}$ , all adjacent laminates of the each optimal laminates in the out-of-plane lamination parameter coordinates are prepared; then FEM analyses are performed ( $F_{nij}$ ).
- (5) If  $\xi F_{RS} > F_{nij}$ , optimality is confirmed.

In this criterion,  $\xi$  is practical tolerant error, and  $\xi$  is 1.005 for the 0.5% error tolerant case. Total number of FEM analyses is 38 when all of these procedures are conducted. The process gives an exact optimal laminate, but it requires a large number of FEM analyses of 1444 ( $= 38 \times 38$ ) in this case that both panel and stiffeners have to be optimized. Moreover, it is required to decide the size of the zoomed region when the zooming is judged to be necessary. Even after the selection of the zoomed region, we have to select a new set of D-optimal laminates from the zoomed region. Since the D-optimal region is usually selected from the laminates at the boundary, it does not always mean that the precise response surface approximation can be obtained around the provisional optimal laminate. It also requires searches of adjacent laminates using the GA. This method is, therefore, not appropriate for searching all-branch method for complicated structures.

#### 4.2. Simplified optimality check for searching all branches

A new simplified optimality-check method is proposed here for the search-all-branches method.

- (1) After obtaining the provisional optimal laminate, a FEM analysis is performed to obtain the true response ( $F_R$ ) of the provisional optimal laminate.
- (2) The error of the response surface of the provisional laminate is calculated, and a radius of allowable error is set to  $\varepsilon$  with considering the error of the response surface. Usually the value of  $\varepsilon$  is set to 5% for approximations of buckling load. All laminates that are superior to the absolute value of the response  $|(1 - \varepsilon)F_R|$  are selected for an estimation of optimality using the global response surface.

For most of the near-optimal points, there are a lot of similar laminates that have similar responses. Therefore, there are usually a lot of selected laminates.

- (3) In the set of the laminates selected before, laminates are classified into groups by focusing only on the in-plane lamination parameters. The laminates that have the same in-plane lamination parameters are collected in the same group.
- (4) Using the global response surface, the top three laminates are selected in each group. When the number of laminates in a group is less than three, all of the laminates are selected in the group. After the selections, FEM analyses of all these selected laminates are performed to obtain the true response of all of these laminates ( $F_n$ ).
- (5) The calculated  $F_n$  is now compared with that of the provisional optimal laminates. When  $\xi F_R < F_n$  is satisfied, it is judged that the optimality is not high enough. After all the comparisons,  $\xi F_R > F_n$  or  $\xi F_R = F_n$  is satisfied, the optimality is confirmed, and the provisional optimal laminate is adopted as a practically optimal laminate.

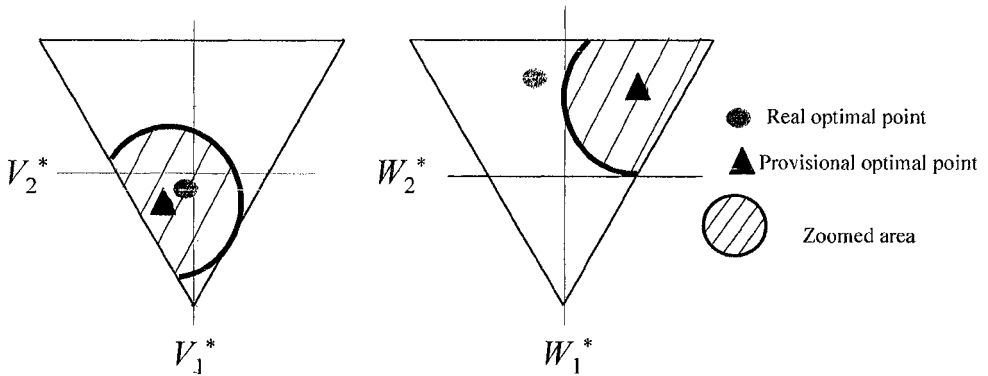
Usually there are quite a lot of adjacent laminates that have the same in-plane lamination parameters and slightly different out-of-plane lamination parameters: those laminates have the same volume fraction of each fiber-angle ply but have different stacking sequences. The result is that the evaluations of high-performance laminates similar to the provisional optimal laminate are just searching adjacent laminates in the out-of-plane lamination parameters and it does not search the adjacent laminates in the in-plane lamination parameters. In the present study, therefore, we classified the set of laminates into the group with focusing only on the in-plane lamination parameters.

This process requires several FEM analyses to obtain true responses, and the number of FEM analyses required depends on the cases. For the stiffened panel in the present study, 22 analyses are required when we consider the both stacking sequences of the stiffeners and of the panel. To reduce the number of analyses, the set of the adjacent laminates judged to be obviously inferior to the provisional optimal laminate with the global response surface can be deleted from the candidate laminates.

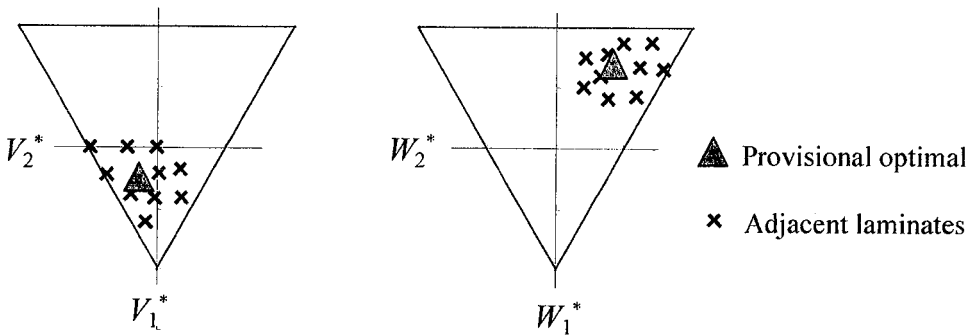
#### 4.3. Modified response surface

For the zooming response surface, it is required to decide the size of the region for zooming, design of experiments and FEM analyses of the selected laminates. However, it is very difficult to decide the region for zooming for the case of the present study: it requires an optimization of the two laminates (the panel and the stiffeners). This difficulty is caused by the fact that the in-plane lamination parameters are not independent of the out-of-plane lamination parameters as described in Fig. 3. In Fig. 3, the provisional optimal point is shown as a solid triangular symbol, and the real optimal point is shown as a solid circular symbol. The real optimal point locates near the provisional optimal point in the in-plane lamination parameters, but





**Figure 3.** Schema of a zoomed response surface in lamination parameter coordinates.



**Figure 4.** Schema of a new modified global response surface in lamination parameter coordinates.

in the out-of-plane lamination parameters the real optimal point locates a little far from the provisional optimal point. When we decide on a circular zooming region as shown in Fig. 3, it may not include the truly optimal point in the out-of-plane lamination parameters. When the zooming region is not appropriate, it may contain few feasible laminates. This makes it difficult to create an appropriate zoomed response surface. These problems can be solved when we adopt a new modified global response surface described in the following.

The concept of the new modified global response surface is shown in Fig. 4. We have a lot of laminates adjacent to the provisional optimal laminate as we already have performed the optimality check proposed before. These adjacent laminates are shown as crosses in Fig. 4. We can include these adjacent laminates into the regression of the global response surface, and that reduces the bias around the provisional optimal laminate in the global response surface. Since the making of the response surface is a simple calculation process of the matrix and vectors, the calculation does not require much computational cost.

As previously mentioned, the D-optimally selected laminates exist at the boundary in the out-of-plane lamination parameters. This causes large bias inside of the space in the out-of-plane lamination parameters. When the provisional optimal laminate

locates inside of the boundary, it may have large bias around the provisional optimal laminate. Adding the adjacent laminates to the provisional optimal laminate drastically reduces the bias around the provisional optimal laminate. This does not require additional FEM analyses and extra design of experiments, and it surely increases the fitness of the response surface just around the provisional optimal laminate.

When the provisional optimal laminate locates at the boundary, adding the adjacent laminates also increases the fitness of the global response surface around the provisional optimal laminate.

## 5. RESULTS AND DISCUSSION

From the FEM analytical results of 90 stacking sequences listed in Table 1, a global response surface is made using a quadratic polynomial. The regressed global response surface is shown in equation (10). In this equation, the variables  $x_1 \sim x_8$  are the replaced variables from the lamination parameters of the panel and the stiffeners described before.

$$\begin{aligned}
 y = & 1187.24 - 155.193x_1 + 69.9832x_2 + 47.4455x_3 + 18.5903x_4 \\
 & + 135.979x_5 + 163.60x_6 - 70.545x_8 - 95.189x_1^2 - 79.636x_2^2 - 39.994x_3^2 \\
 & - 52.6579x_4^2 + 164.78x_5^2 - 170.168x_6^2 - 359.847x_7^2 + 108.913x_8^2 \\
 & - 90.2156x_1x_5 + 35.067x_1x_6 + 63.638x_1x_7 + 58.032x_1x_8 + 26.295x_2x_3 \\
 & + 36.109x_2x_5 - 32.573x_2x_6 - 31.2883x_2x_7 - 38.641x_3x_7 - 21.108x_3x_8 \\
 & - 96.1782x_5x_6 - 232.551x_5x_7 + 98.272x_5x_8 - 64.8719x_7x_8. \quad (10)
 \end{aligned}$$

The adjusted coefficient of the multiple determination ( $R_{ad}^2$ ) of equation (10) is 0.9678, and the value of  $R_{ad}^2$  seems to show good fitness.

Using the search-all-branches technique, the provisional optimal laminate is obtained as follows.

$$\begin{aligned}
 \text{Stiffener: } & [0/0/90/90]_s & \text{Panel: } & [90/0/45/-45]_s \\
 \text{Buckling load } y = & 1273.4 \text{ N}
 \end{aligned}$$

The true buckling load of this provisional optimal laminate is calculated with FEM analysis; the FEM result is 1291.1 N, and the error of the buckling load is  $-1.37\%$ . The allowable error  $\varepsilon$  is set to  $5\%$  here. The adjacent laminates are collected by searching all branches within the allowable error. The total number of adjacent laminates is 22, and the all of these laminates are shown in Table 2 with the results of the global response surface and the results of the FEM analyses. In Table 2, the laminates with solid circles show the higher buckling load than the provisional optimal laminate. This means the optimality of the provisional optimal laminate is rejected.

A new modified global response surface is made using the additional 23 FEM analyses: FEM results of the 22 adjacent laminates and the FEM result of the

**Table 2.**  
Stacking sequences and results of FEM analyses of adjacent laminates

	Plate	Stiffener	RS value (N)	FEM value (N)
1	[90/0/0/90]s	[0/0/90/90]s	1269	● 1318.6
2	[90/0/0/90]s	[0/90/45/−45]s	1268.6	1286.3
3	[90/0/45/−45]s	[0/90/0/90]s	1264.8	1269.5
4	[90/0/0/90]s	[0/90/0/90]s	1263.5	● 1297.7
5	[90/0/0/90]s	[0/90/90/0]s	1256.5	1285.8
6	[90/0/45/−45]s	[0/90/90/0]s	1256.1	1256.8
7	[90/0/90/0]s	[0/0/90/90]s	1255.1	● 1316
8	[90/0/45/−45]s	[90/0/90/90]s	1251.1	1269.5
9	[90/0/0/90]s	[90/0/45/−45]s	1249.7	1254.1
10	[90/0/90/0]s	[0/90/0/90]s	1246.8	1291
11	[90/0/90/0]s	[0/90/45/−45]s	1245.4	1270.9
12	[0/90/0/0]s	[0/90/45/−45]s	1240.5	1248.6
13	[0/90/90/0]s	[0/90/45/−45]s	1240.2	1243.8
14	[90/0/90/0]s	[0/90/90/0]s	1238.4	1276.4
15	[0/90/0/0]s	[90/0/45/−45]s	1234.5	1222.7
16	[90/0/45/−45]s	[0/90/45/−45]S	1233.6	1239.9
17	[0/90/0/0]s	[0/90/0/90]s	1232.7	1254.2
18	[90/0/0/90]S	[0/45/−45/90]s	1230.3	● 1298.6
19	[0/90/0/0]s	[0/90/90/0]s	1230	1245.1
20	[0/90/0/0]s	[0/0/90/90]s	1229.7	1270.3
21	[0/90/90/0]s	[90/0/45/−45]s	1229.4	1216.6
22	[90/0/0/90]s	[90/0/90/90]s	1228.9	1263.5

provisional optimal laminates. The obtained modified global response surface is shown in equation (11). The adjusted coefficient of the multiple determination ( $R_{ad}^2$ ) of equation (10) is 0.9746, and the value of  $R_{ad}^2$  seems to be better fitted.

$$\begin{aligned} y = & 1194.26 - 182.147x_1 + 69.8994x_2 + 78.003x_3 + 10.6713x_4 + 136.298x_5 \\ & + 176.1304x_6 - 72.1239x_8 - 104.669x_1^2 - 74.1663x_2^2 - 28.3603x_3^2 \\ & + 48.2736x_4^2 + 104.5289x_5^2 - 170.103x_6^2 - 391.387x_7^2 + 111.3269x_8^2 \\ & + 63.75196x_1x_2 - 43.5825x_1x_4 - 90.161x_1x_5 + 35.361x_1x_6 \\ & + 66.7683x_1x_7 + 53.6066x_1x_8 + 41.5842x_2x_5 - 29.4642x_2x_6 - 36.8058x_2x_7 \\ & - 42.5601x_3x_7 - 15.7445x_3x_8 - 101.513x_5x_6 - 159.808x_5x_7 \\ & + 104.6351x_5x_8 - 67.5156x_7x_8 \end{aligned} \tag{11}$$

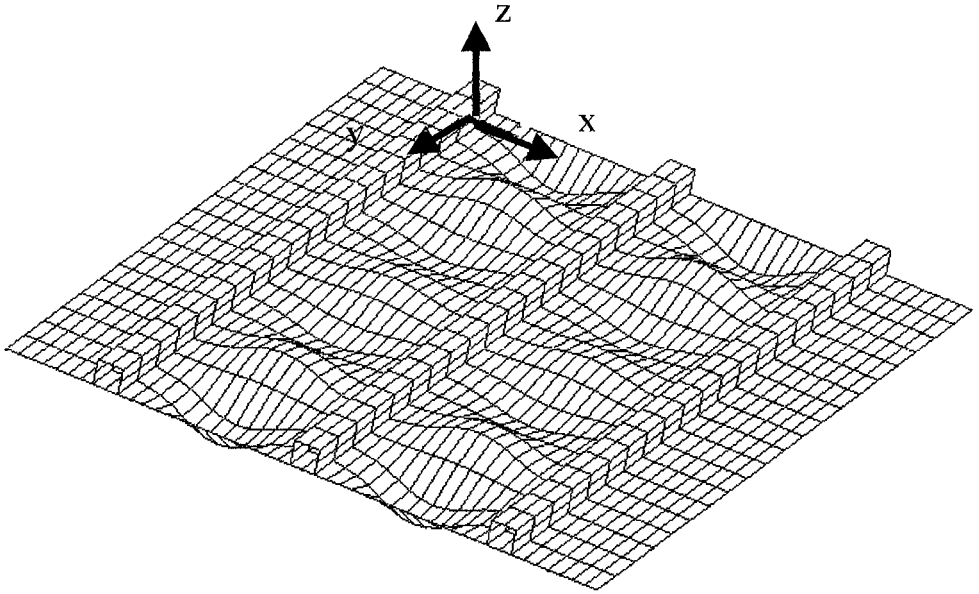
The optimal laminate is obtained with the modified global response surface using the searching all branches. The obtained optimal laminate is as follows.

Stiffener: [0/0/90/90]s

Panel: [90/0/0/90]s

Buckling load  $y = 1307.2$  N

The true buckling load of this provisional optimal laminate is calculated with FEM analysis; the FEM result is 1318.6 N, and the error of the buckling load is only −0.86%. The buckling mode is shown in Fig. 5. The buckling waves are



**Figure 5.** Buckling mode of the obtained optimal laminate.

created between in the panel between the stiffeners. From the similar optimality check, the laminate is proved to be the real optimal laminate. In addition, to confirm the effectiveness of the optimality check method, we performed many FEM analyses around the optimal laminate, and we confirmed that it was the truly optimal laminate.

In order to confirm the reduction of the bias around the optimal laminate, residual errors of the response surfaces are compared with the two global response surfaces. For the comparisons, residual errors of all of the laminates ( $113 \text{ laminates} = 90 + 23$ ) used for regression are calculated and the residual errors of the top 30 laminates obtained with the search-all-branches method using the modified global response surface are also calculated here. In Table 3, the comparisons of the FEM analyses of the top 30 laminates are shown. In the table, GRS means the errors of the responses of the global response surface, and MRS means the errors of the responses of the modified global response surface. For the initial global response surface, average error of all the laminates used for the regression is 3.48%, and the average error of the top 30 laminates is 3.06%. For the modified global response surface, average error of the all laminates used for the regression is 3.31%, and the average error of the top 30 laminates is 1.19%. As shown in Table 3, the maximum error in the top 30 laminates is only 3.54%. These reveal that the bias around the optimal laminate is surely reduced to one-third. However, the error of the entire design space is not drastically reduced. This means that the good fitness of the response is maintained for the entire space and the bias around the optimal laminate is drastically improved.

**Table 3.**

Error of buckling load for the best thirty laminates

	Plate	Stiffener	FEM	GRS (%)	MRS (%)
1	[90/0/0/90]s	[0/0/90/90]s	1318.647	3.761947	0.865797
2	[90/0/90/0]s	[0/0/90/90]s	1315.979	4.628439	1.860158
3	[90/0/0/90]s	[0/45/−45/90]s	1298.607	5.259236	2.001129
4	[90/0/0/90]s	[0/90/0/90]s	1297.703	2.639495	0.373183
5	[90/0/45/−45]s	[0/0/90/90]s	1291.148	1.352145	−0.20306
6	[90/0/90/0]s	[0/90/0/90]s	1290.984	3.42407	1.305531
7	[90/0/0/90]s	[0/90/45/−45]s	1286.28	1.376023	−0.37165
8	[90/0/0/90]s	[0/90/90/0]s	1285.772	2.280482	0.240463
9	[90/0/90/0]s	[0/45/−45/90]s	1285.278	5.925425	2.862275
10	[90/0/90/0]s	[0/90/90/0]s	1276.369	2.973968	1.089716
11	[90/0/90/0]s	[0/90/45/−45]s	1270.885	2.008404	0.488203
12	[90/0/0/90]s	[0/0/0/90]s	1270.832	5.078742	2.190084
13	[0/90/0/0]s	[0/0/90/90]s	1270.304	3.19718	0.836327
14	[90/0/45/−45]s	[0/90/0/90]s	1269.49	0.371811	−0.54588
15	[90/0/45/−45]s	[90/0/90/90]s	1269.466	1.450658	0.842528
16	[90/0/90/0]s	[0/0/0/90]s	1267.697	6.291502	3.545579
17	[90/0/0/90]s	[90/0/90/90]s	1263.486	2.736587	0.759517
18	[90/0/0/90]s	[0/0/90/0]s	1261.404	4.21941	1.765002
19	[0/90/90/0]s	[0/0/90/90]s	1260.913	3.077396	0.378559
20	[90/0/90/0]s	[0/0/90/0]s	1259.651	5.650057	3.354977
21	[0/90/0/0]s	[0/45/−45/90]s	1259.639	5.068796	2.28387
22	[90/0/0/90]s	[90/0/0/90]s	1257.996	1.908281	0.131646
23	[90/0/45/−45]s	[0/90/90/0]s	1256.832	0.059013	−0.63082
24	[0/90/90/0]s	[0/45/−45/90]s	1255.174	4.568602	1.341957
25	[0/90/0/0]s	[0/90/0/90]s	1254.208	1.713265	−0.04561
26	[90/0/0/90]s	[90/0/45/−45]s	1254.108	0.352314	0.204002
27	[90/0/45/−45]s	[0/45/−45/90]s	1250.341	4.202933	2.045922
28	[0/90/0/0]s	[0/90/45/−45]s	1248.6	0.64791	−0.62552
29	[90/0/90/0]s	[90/0/90/90]s	1248.397	2.872267	1.05794
30	[0/90/45/−45]s	[0/0/90/90]s	1245.478	2.654265	1.513338

The buckling load error of the truly optimal laminate is only  $-0.86\%$ . This is practically acceptable preciseness, and other FEM analyses for optimality check are not needed practically.

This modified global response surface is not limited to the search-all-branches method, and the modified global response surface is applicable to other searching methods like GA. This method is on the basis of the assumption that the initial global response surface gives approximately good fitness. When the provisional optimal laminate locates near the real optimal laminate and the real optimal laminate exists in the set of the laminates prepared for the optimality check, it is easy to say that the real optimal can be obtained with this method. Actually, the truly optimal laminate is the top laminate prepared for the optimality check in Table 1. It is, however, empirically revealed that the error of the provisional optimal laminate is less than  $5\%$ , even for the initial global response surface for the buckling load

maximization problems [13, 16]. This means that even the initial global response surface gives good fitness. Moreover, even for the inferior laminates in Table 2, the error of the modified global response surface is very small. For example, the average error of the top 100 laminates of the initial global response surface is 3.05%, and that of the modified response surface is 1.90%. This means that even outside of the adjacent laminates, the bias of the laminates is reduced using the new modified global response surface.

From these, it is possible to obtain the truly optimal laminate using the modified global response surface method without using additional zooming.

## 6. CONCLUSIONS

In the present study, a new simpler optimality check method for the search-all-branches method is proposed, and a new modified global response surface that does not require zooming is explained in detail here. This method is applied to a stiffened panel with hat-type stiffeners for maximizations of the buckling load. The results obtained are as follows.

- (1) The simple optimality check method is appropriate for the search-all-branches method.
- (2) The modified global response surface obtained by adding the FEM analyses into the regression of the global response surface drastically reduces the bias around the optimal laminate, and it gives the truly optimal laminate without designing complicated zooming.

## REFERENCES

1. M. Miki, Design of laminated fibrous composite plates with required flexural stiffness, *ASTM STP* **864**, 387–400 (1985).
2. H. Fukunaga and T. W. Chou, Simplified design techniques for laminated cylindrical pressure vessels under stiffness and strength constraints, *J. Compos. Mater.* **22**, 1156–1169 (1988).
3. I. Susuki, Strength optimization of multidirectional laminates in an in-plane stress state, *Mater. Sci. Engng A* **143**, 175–185 (1991).
4. R. Le Riche and R. T. Haftka, Optimization of laminate stacking sequence for buckling load maximization by genetic algorithm, *AIAA J.* **31** (5), 951–956 (1993).
5. A. Todoroki, K. Watanabe and H. Kobayashi, Application of genetic algorithms to stiffness optimization of laminated composite plates with stress-concentrated open holes, *JSME Intern. J. Series A* **38** (4), 458–464 (1995).
6. H. Fukunaga and H. Sekine, Stiffness characteristics and their design method of symmetric laminates, *J. Japan Soc. Aeronautical Space Sci.* **42** (485), 371–380 (1994) (in Japanese).
7. K. Yamazaki, Two-level optimization technique of composite laminate panels by genetic algorithms, AIAA Paper 96-1539, in: *Proc. 37th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics and Materials Conference*, Salt Lake City, UT, pp. 1882–1887 (1996).
8. P. N. Harrison, R. Le Riche and R. T. Haftka, Design of stiffened composite panels by genetic algorithm and response surface approximations, AIAA Paper 95-1163, in: *Proc. 36th*

- AiAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference, New Orleans, LA, Part 1, pp. 58–68 (1995).
9. M. Zako, N. Takano and N. Takeda, Cost effective design procedure for laminated composite structure based on GA, *Sci. Engng Compos. Mater.* **6** (2), 131–140 (1997).
  10. S. N. Grangadharan, S. Nagendra and Y. Fiocca, Response surface based laminate stacking sequence optimization under stability constraints, AIAA Paper 97-1236, in: *Proc. 38th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics and Material Conference*, Kissimmee, FL, Part 3, pp. 2381–2389 (1997).
  11. Y. Aoki and O. I. Byon, Buckling design of FRP laminated cylindrical shells by use of evolutionary algorithm, in: *Proc. 1998 ASME/JSME Joint Pressure Vessels and Piping Conference*, San Diego, CA, Finite Element Applications: Liner, Non-linear, Optimization and Fatigue and Fracture, PVP-370, p. 51–56 (1997).
  12. Y. Ohta and R. T. Haftka, Optimization of FRP laminated plates under uncertainty by fuzzy-set and genetic algorithm, AIAA Paper 97-1231, in: *Proc. 38th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics and Material Conference*, Kissimmee, FL, Part 2, pp. 1325–1330 (1997).
  13. A. Todoroki and R. T. Haftka, Lamination parameters for efficient genetic optimization of the stacking sequences of composite panel, in: *Proc. 7th AIAA/USAF/NASA/ISSMO MAO*, St. Louis MO, AIAA Paper 98-4816, pp. 870–889 (1998).
  14. A. Todoroki and R. T. Haftka, Stacking sequence optimization by a genetic algorithm with a new recessive gene like repair strategy, *Composites Part B* **29B**, 277–285 (1998).
  15. A. Todoroki and M. Sasai, Improvement of design reliability for buckling load maximization of composite cylinder using genetic algorithm with recessive-gene-like repair, *JSME Intern. J. Series A* **42** (4), 530–536 (1999).
  16. A. Todoroki and M. Sasai, Stacking sequence optimizations using GA with zoomed response surface on lamination parameters, *Adv. Composite Mater.* **11** (3), 299–318 (2003).
  17. M. P. Nemeth, Importance of anisotropy on buckling of compression-loaded symmetric composite plates, *AIAA J.* **24** (11), 1831–1835 (1986).
  18. R. T. Haftka and Z. Gürdal, *Elements of Structural Optimization, Solid Mechanics and its Applications*. Kluwer Academic Publications, Dordrecht (1993).
  19. Z. Gürdal, R. T. Haftka and P. Hajela, *Design and Optimization of Laminated Composite Materials*. Wiley InterScience (1999).
  20. H. Fukunaga, Recent research on optimization of composite structures, *Proc. OPTIS '98, JSME* **98** (14), 65–70 (1998) (in Japanese).
  21. R. H. Myers and D. C. Montgomery, *Response Surface Methodology: Process and Product Optimization Using Designed Experiments*. John Wiley, New York (1995).
  22. A. Todoroki and T. Ishikawa, Design of experiments for stacking sequence optimizations with genetic algorithm using response surface approximation, *Composite Structures* (to be published).